shape of real river meanders [*see illustration on next page*]. At the axis of bend the channel is directed in the mean down-valley direction and the angle of deflection is zero, whereas at the point of inflection the angle of deflection reaches a maximum value.

A sine-generated curve differs from a sine curve, from a series of connected semicircles or from any other familiar geometric curve in that it has the smallest variation of the changes of direction. This means that when the changes in direction are tabulated for a given distance along several hypothetical meanders, the sums of the squares of these changes will be less for a sine-generated curve than for any other regular curve of the same length. This operation was performed for four different curves of the same length, wavelength and sinuosity—a parabolic curve, a sine curve, a circular curve and a sine-generated curve—in the illustration on page 65. When the squares of the changes in direction were measured in degrees over 10 equally spaced intervals for each curve, the resulting values were: parabolic curve, 5,210; sine curve, 5,200; circular curve, 4,840; sine-generated curve, 3,940.

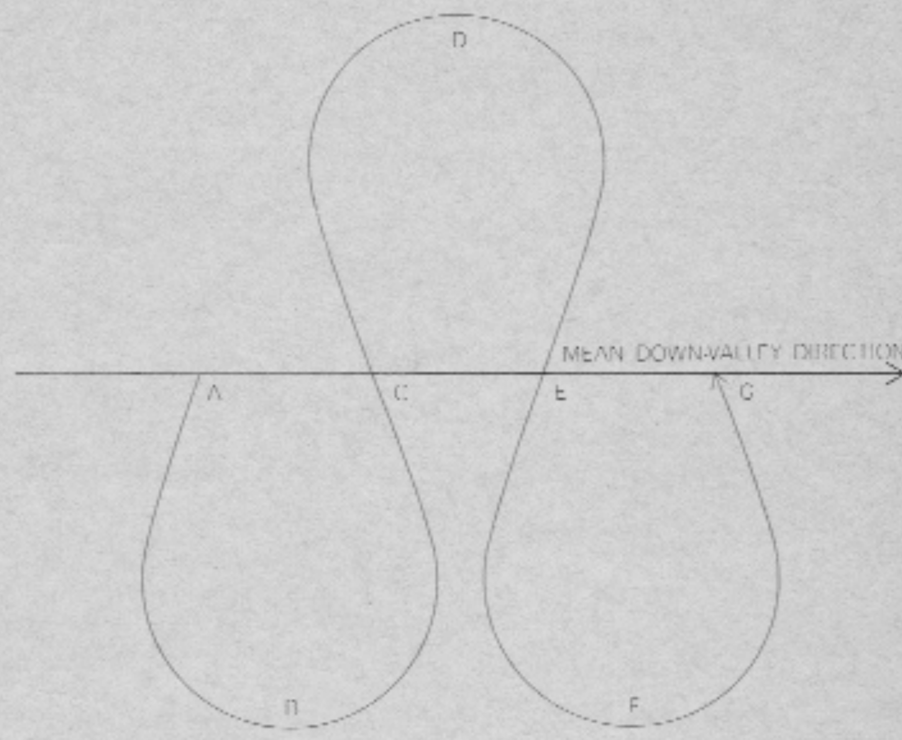
Curve of Minimum Total Work

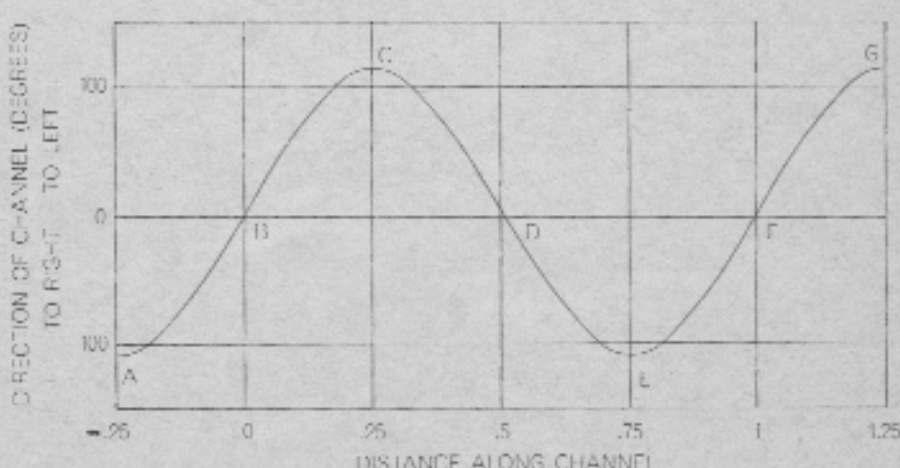
Another property closely associated with the fact that a sine-generated curve minimizes the sum of the squares of the changes in direction is that it is also the curve of minimum total work in bending. This property can be demonstrated by bending a thin strip of spring steel into various configurations by holding the strip firmly at two points and allowing the length between the fixed points to assume an unconstrained shape [*see top illustration on pages 66 and 67*]. The strip will naturally avoid any concentration of bending and will assume a shape in which the bend is as uniform as possible. In effect the strip will assume a shape that minimizes total work, since the work done in each element of length is proportional to the square of its angular deflection. The shapes assumed by the strip are sine-generated curves and indeed are good models of river meanders.

A catastrophic example of a sine-generated curve on a much larger scale was provided by the wreck of a Southern Railway freight train near Greenville, S.C., on May 31, 1965 [*see bottom illustration on page 67*]. Thirty adjacent flatcars carried as their load 700-foot sections of track rail chained in a bundle to the car beds. The train, pulled by five locomotives, collided with a bulldozer and was derailed. The violent compressive strain folded the trainload of rails into a drastically foreshortened snakelike configuration. The elastic properties of the steel rails tended to minimize total bending exactly as in the case of the spring-steel strip, and as a result the wrecked train assumed the shape of a sine-generated curve that distributed the bending as uniformly as possible. This example is particularly appropriate to our discussion of river meanders because, like river meanders, the bent rails deviate in a random way from the perfect symmetry of a sine-generated curve while preserving its essential form.

The Shaping Mechanism

The mechanism for changing the course of a river channel is contained





SINE-GENERATED CURVE (*top*) **closely approximates the shape of real river meanders. This means that the angular direction of the channel at any point with respect to the mean down-valley direction** (*toward the right*) **is a sine function of the distance measured along the channel** (*graph at bottom*). **At the axis of each bend** (*B, D and F*) **the channel is directed in the mean down-valley direction and the angle of deflection is zero, whereas at each point of inflection** (*A, C, E and G*) **the angle of deflection reaches a maximum value.**