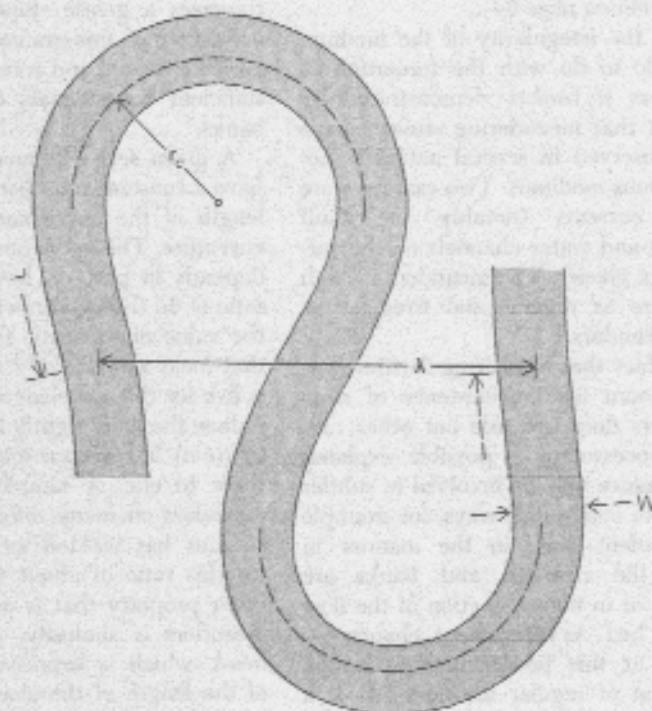


WIDTH OF CHANNEL ( $W$ ) = 1  
 WAVELENGTH ( $\lambda$ ) = 11.5  
 LENGTH OF CHANNEL ( $L$ ) = 18.5  
 RADIUS OF CURVATURE ( $r_c$ ) = 2.3



WIDTH OF CHANNEL ( $W$ ) = 1  
 WAVELENGTH ( $\lambda$ ) = 6.9  
 LENGTH OF CHANNEL ( $L$ ) = 24.8  
 RADIUS OF CURVATURE ( $r_c$ ) = 2.3

PROPERTIES used to describe river meanders are indicated for two typical meander curves. A series of meanders has a regular appearance on a map whenever there tends to be a constant ratio between the wavelength ( $\lambda$ ) of the curve and its radius of curvature ( $r_c$ ). The value of this ratio for the meander that looks rather like a sine wave (top) is five to one; the more tightly looped meander (bottom) has a corresponding value of three to one. An average value for this ratio is about 4.7 to one. Sinuosity, or tightness of bend, is expressed as the ratio of the length of the channel ( $L$ ) in a given curve to the wavelength of curve. The value of this ratio for the top curve is 1.4 to one and for the bottom curve 3.6 to one. On the average the value of this ratio ranges between 1.3 to one and four to one.

and maps that accompany this article will show that typical river meanders do not exactly follow any of the familiar curves of elementary geometry. The portion of the meander near the axis of bend (the center of the curve) does resemble the arc of a circle, but only approximately. Neither is the curve of a meander quite a sine wave. Generally the circular segment in the bend is too long to be well described by a sine wave. The straight segment at the point of inflection—the point where the curvature of the channel changes direction—prevents a meander from being simply a series of connected semicircles.

### Sine-generated Curves

We first recognized the principal characteristics of the actual curve traced out by a typical river meander in the course of a mathematical analysis aimed at generating meander-like curves by means of "random walk" techniques. A random walk is a path described by successive moves on a surface (for example a sheet of graph paper); each move is generally a fixed unit of distance, but the direction of any move is determined by some random process (for example the turn of a card, the throw of a die or the sequence of a table of random numbers). Depending on the purpose of the experiment, there is usually at least one constraint placed on the direction of the move. In our random-walk study one of the constraints we adopted was that the path was to begin at some point  $A$  and end at some other point  $B$  in a given number of steps. In other words, the end points and the length of the path were fixed but the path itself was "free."

The mathematics involved in finding the average, or most probable, path taken by a random walk of fixed length had been worked out in 1951 by Hermann von Schelling of the General Electric Company. The exact solution is expressed by an elliptic integral, but in our case a sufficiently accurate approximation states that the most probable geometry for a river is one in which the angular direction of the channel at any point with respect to the mean down-valley direction is a sine function of the distance measured along the channel [see illustration on opposite page].

The curve that is traced out by this most probable random walk between two points in a river valley we named a "sine-generated" curve. As it happens, this curve closely approximates the