Mathematically Meaningful Mistakes
Does 7 times 13 equal 28?
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## The Lesson Scenario-More Detail

Teacher writes on board: $13 \times 7=28$

This is an easy, straightforward problem. Most kids will (or should) come up with 91. But, many of the students will answer the problem without true understanding of how to multiply: they learned only the procedures of multiplication. We all remember the famous or rather infamous. : "take down, carry, and put an x or zero under" expressions that "teach" multiplication without meaning concentrating on nothing else but on the process.

Teachers have to make sure that students truly master multiplication (emphasizing place values) at the time when it is first introduced to them. Students will be forever lost in mathematics without the encompassing knowledge of this seemingly simple concept. The lack of mastery of multiplication will make comprehending division and the operations with fraction almost impossible. Let's look even further: how can teachers introduce percents, the multiplication of algebraic expressions, or exponents to a student who practices math in a mechanical way but has no number sense?

Even when our students answer the above multiplication problem correctly, instead of giving the usual response of "that's right", the teacher should probe the students with "Are you sure? I have some evidence to the contrary. Let me show you this video."


This skit can be seen as an example of how a teacher can misunderstand a student's knowledge and fail to make use of learning opportunities. In the hands of the comic masters, it is funny; in the classroom, it is often distressing.

Students will enjoy this "ancient" parody. Most of them will catch Lou's mistake of putting the 7 in the wrong place. Again, that might only be a recalling of the procedures they were taught, not real understanding. What if a student forgets or mixes up a step in the process? That is what happened to Lou, and it can happen to our students.

Our goal is to help our pupil become multiplication masters. This skit could be used productively with fourth and fifth grade children. After showing the video the teacher asks: "See 7 times 13 can be 28. Isn't that right?"

As we said earlier, a common answer to this question from student's point of view is: "Lou is not putting the 7 in the "right" place. He or she will usually say that the 7 should go under the 2.


Why figure 2 not figure 1 ?
Let's revisit figure 1 in more detail:

$$
\begin{array}{r}
13 \\
\times 7 \\
\hline
\end{array}
$$

Step 1 : multiply $3 \times 7$.

$$
\begin{array}{r}
3 \\
\times 7 \\
\hline 21 \\
\hline
\end{array}
$$

Step 2: multiply $1 \times 7$. But where do we put the 7 ?


## 7 ?

Let's rethink the problem again. We are multiplying 13 times 7. Is that the same as multiplying $3 \times 7$ followed by $1 \times 7$ and adding the answers? No, that's the Costello method.

How do we say 13 with meaning?
13 is the same as $10+3$


NOT $1+3$.
So we can rewrite the problem as

$$
\begin{array}{r}
10+3 \\
\times 7
\end{array}
$$

and do the respective multiplications:

$$
\begin{array}{r}
10+3 \\
\times 7 \\
3 \times 7=21 \\
10 \times 7=70 \\
\hline 91
\end{array}
$$

## Visual Model $7 \times 4=28$ Donuts

The video actually provides a visual model that can, potentially, be used to solve the problem. In the first 10 seconds, another sailor (played by Shemp Howard of the Three Stooges) hands Lou a tray of donuts. There are 7 rows of 4 donuts (a total of 28 which is 7 x 4 ). Using this visual model (the tray of donuts in the video), Bud could have shown Lou that 7 times 4 must be 28 , and that each of the seven officers would receive four donuts. Here's a model of the tray.


There are 4 rows of 7 doughnuts each which models nicely that $7 \times 4=28$. However, when Lou makes the claim that $13 \times 7=28$, there is no visual model to confirm it. What follows is a visual model of what $13 \times 7$ really equals.

Visual Model of 13x7=91 - Addition method
Here we have 13 donuts .

## -००००००००००००

Showing 7 rows of 13 donuts confirms that we have 91 donuts by simple counting.


In the addition model

\section*{| 13 |
| :--- |
| 13 |
| 13 |
| 13 |
| 13 |
| 13 |
| 13 |}

Bud counts by 3s which makes perfect sense until he switches to counting the ones as just an additional doughnut like this.


Here's the switch. Lou takes the 1 and counts it as just 1 doughnut


Instead adding just one doughnut (in the previous step) he should be adding ten doughnuts (see figure below) because 13 means $10+3$.


Here's a picture of the revised adding scheme as Lou should have done it.


If you rearrange the doughnuts above like this (Fig. below)


Or this one?

$10 \times 10$ donut tray

$10 \times 10$ donut tray

This can help student's understand why the 7 should be written as 70 in the multiplication algorithm to emphasize the place value of the 7 .

Since numbers are masters of disguise students must become experts at uncloaking them. For example, here are some forms of 13 that children have experienced (by grade 2.)


Since Figure 4 is the one students start to use all the time, they tend to forget that the 1 in 13 is really just a disguise or shortcut for a "a stack of 10 " or just plain ole 10.

## How can we help Bud teach this well?

Since we want to turn this video into a significant learning experience for your students, a good follow up is to ask the students for suggestions on how Bud could take advantage of this moment to teach Lou about place value. One possibility would be for Bud to refer back to the pan of doughnuts that models $7 \times 4=28$ and show a pan of 7 rows of 13 doughnuts which would clearly cause some cognitive dissonance for Lou when he looks at the algorithm that he just wrote on the blackboard showing $13 \times 7=28$ where the 7 is placed under the one.


Note: It would be a better comic if Homer (Lou?) is scratching his head. Or even better the one below.

$13 \times 7=28$ ?

## Epilogue/Closure

If teachers consider mathematics to be nothing more than a list of facts to be memorized, students will be as confused as Lou Costello, and sadly, no one is laughing at that. In the classroom, miscommunications can hurt students.

If teachers fall short of explaining the underlying implications of math problems, than students will feel lost in an ocean of contentious nonsense. If teachers keep repeating "Who is on first," without addressing who is who; comedy will turn into academic tragedy.

Mathematics is part of our everyday life; it is the language, the underlying foundation of everything that surrounds us, the groundwork of nature and all human scientific discoveries and inventions. Wherever we turn we encounter the fruits of mathematics: cars, houses, computers, even IPADs, cell phones, and credit cards.

Many students think it is wonderful that math improved the quality of modern life with pleasing technological trinkets, but rarely see the connection between human advancement and their own math education. Students lose interest in math (and in education in general) as early as in third or fourth grade and see school math as a boring waste of time. They advance from grade to grade confused and lost in the mathematical procedures that are taught to them without meaning. There is no humor in that.

We mathematics and science teachers are not speaking nonsense. We are discussing very real concepts: thoughts that can become powerful tools.

## References

1. Berkman, Robert. "Teacher as Kimp", (Arithmetic Teacher, NCTM: Feb. 1994) p. 326 http://dmcpress.org/cm/acmultiply/Teacheraskimp.pdf
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3. Shemp Howard, Abbott and Costello, " 7 X 13 is 28 " routine. https://www.youtube.com/watch?v= HvGven4qJ0
